Chemistry 240 Semester 01-2009
Homework for Submission #1 – Answer Key

1) There are various ways of expressing the speed of the molecules of a gas. The root mean square speed has been dealt with in class. The average speed of the molecules of a gas is given by $\sqrt{\frac{8RT}{\pi M}}$ and the most probable speed is given by $\sqrt{\frac{2RT}{M}}$.

a) Prove that the root mean square speed is the largest and the most probable speed the smallest of these quantities, regardless of the value of T or M.

$$u_{rms} = \sqrt{\frac{3RT}{M}},\ u_p = \sqrt{\frac{2RT}{M}},\ \bar{u} = \sqrt{\frac{8RT}{\pi M}}$$

Hence

$$u_{rms} : u_p : \bar{u} = \sqrt{\frac{3RT}{M}} : \sqrt{\frac{2RT}{M}} : \sqrt{\frac{8RT}{\pi M}} = \sqrt{3}\sqrt{\frac{RT}{M}} : \sqrt{2}\sqrt{\frac{RT}{M}} : \sqrt{\frac{8}{\pi}}\sqrt{\frac{RT}{M}} = \sqrt{3} : \sqrt{2} : \sqrt{\frac{8}{\pi}} = 1.732:1.414:1.596$$

but $1.732 > 1.596 > 1.414$ and so

$$\sqrt{\frac{3RT}{M}} > \sqrt{\frac{8RT}{\pi M}} > \sqrt{\frac{2RT}{M}}$$

$$\therefore u_{rms} > \bar{u} > u_p$$

b) Evaluate these speeds for nitrogen gas at 100°C.

$$\sqrt{\frac{RT}{M}} \text{ for } N_2 \text{ at 373K} = \sqrt{\frac{8.31\times 373}{28\times 10^{-3}}} = 3.327\times 10^2$$

$$\therefore u_{rms} = 1.732\times 3.327\times 10^2 = 5.76\times 10^{-2}\, ms^{-1}$$

$$\bar{u} = 1.596\times 3.327\times 10^2 = 5.31\times 10^2\, ms^{-1}$$

$$u_p = 1.414\times 3.327\times 10^2 = 4.71\times 10^2\, ms^{-1}$$

c) Compare your value for nitrogen with the escape velocity for the Earth of 11.2 km s^{-1} and comment on your result.

These values are all much smaller than the escape velocity. Consequently the Earth has very little tendency to lose nitrogen from its atmosphere and this helps to explain why the Earth's atmosphere is so rich in nitrogen today.

2) You are told that 2.55 g of a gaseous hydrocarbon occupies a vessel of volume 3.00 L at 0.950 atm and 82.0°C. Draw the Lewis structure for this hydrocarbon.

$PV = nRT$

$\therefore PV = \frac{m}{M} RT$

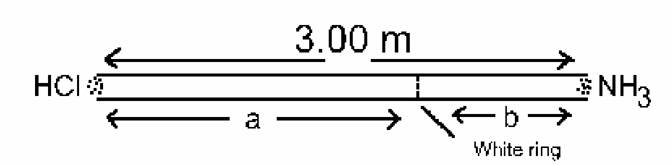
$\therefore M = \frac{mRT}{PV} = \frac{2.55 \times 0.0821 \times (273 + 82)}{0.950 \times 3.00} = 26.1 \text{ to 3 sig. figs.}$

Hence the hydrocarbon must be C_2H_2 and the Lewis structure

H-C≡C-H

3) Ammonia and hydrogen chloride gases react on contact to form a white solid. A tube is plugged with glass wool soaked in concentrated hydrochloric acid at the left hand end and another plug soaked in concentrated ammonia at the right hand end at the same time. The distance between the inside surfaces of the plugs is 3.00 m. After a while a narrow white annular band has formed on the inside of the tube, perpendicular to its length. Calculate the distance of the band from the left hand end of the tube.

The situation is as follows. We have to calculate a in the diagram below.



After time t, HCl has diffused distance $V_L t$ and after the same time, NH_3 has diffused distance $V_R t$. Since t is the time that the gases take to meet and form the white ring, they are the same in each case.

$\text{Rate of diffusion } \alpha \frac{1}{\sqrt{M}}$

$\frac{V_L t}{V_R t} = \frac{V_L}{V_R} = \frac{a}{b}$

$\text{But } \frac{V_L}{V_R} = \text{relative rates of diffusion and since}$

$\text{rate of diffusion } \alpha \frac{1}{\sqrt{M}} \text{ and so we have that } \frac{V_L}{V_R} = \sqrt{\frac{M_{HCl}}{M_{NH_3}}} = \sqrt{36.5/17.0} = 1.46528$

But a = 3.00 - b

$\therefore \frac{a}{b} = \frac{3.00 - b}{b} = 1.46528$

$\therefore b = 1.2169 \text{ m and } a = 1.7830 \approx \underline{\underline{1.78 \text{ m}}} \text{ to 3 sig. figs.}$

4) Two glass vessels, one containing oxygen at 4.00 atm and the other containing nitrogen at 5.25 atm. Given that the volume of the vessels is 1.00 L and 2.00 L

respectively, calculate the final pressure if the two vessels are connected without any change in total volume or temperature.

The gases may be considered separately. In each case the final volume is 1 L + 2 L = 3 L. Since the temperature is constant we can applies Boyle's law which may be stated:

$$P_2 = P_1 \frac{V_1}{V_2}$$

For oxygen we have P_{O2} = 4 x 1/3 = 1.33 atm.

For nitrogen we have P_{N2} = 5.25 x 2/3 = 3.5 atm

Thus, by Dalton's law of partial pressures, the total pressure of the final mixture is 1.33 + 3.50 = <u>4.83 atm</u>

Now suppose that the gases are hydrogen chloride and ammonia respectively, what is the final pressure after connection and cooling to the original temperature? (NB the volume of solids is negligible compared to that of gases.)

Here a reaction occurs:

$HCl(g) + NH_3(g) \rightarrow NH_4Cl(s)$

Adjusted to 1 atm by Boyle's law, the volume of HCl is 1 x 4/1 = 4 L and the volume of ammonia is 2 x 5.25/1 = 10.5 L

Hence, by Gay-Lussac's lay of combining volumes (an extension of Avogadro's law i.e. $V \alpha\ n$), 4 L of NH_3 combine with 4 L of HCl, leaving 10.5-4 = 6.5 L of NH_3 uncombined. This is confined to the 3 L container and so its pressure is given, again by Boyle's law, by
1 x 6.5/3 = <u>2.17 atm</u>

5) A vessel contains 46.2 g of a gas. The vessel and its contents are heated from 0.00°C to 327.2°C. What mass of gas must be allowed to escape to maintain a constant pressure? (Assume the volume of the container remains constant.)

$$PV = nRT = \frac{m}{M}RT$$

$$\therefore m \alpha \frac{1}{T} \text{ at constant M, P and V}$$

$$\therefore \frac{m_1}{m_2} = \frac{T_2}{T_1}$$

$$\therefore m_2 = m_1 \frac{T_1}{T_2} = 46.2 \times \frac{273}{327.2 + 273} = 19.548 = 19.5g \text{ to 3 sig. figs.}$$

Hence the mass escaping = 46.2 g – 19.5 g = <u>26.7 g</u>